Eur. Phys. J. B (2006)
DOI: 10.1140/epjb/e2006-00451-y

THE EUROPEAN
PHYSICAL JOURNAL B

Segregation effect in symmetric cellular automata model for two-lane mixed traffic

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Received 29 June 2006 / Received in final form 16 November 2006
Published online 22 December 2006 –

Abstract. Segregation effects commonly exist in granular mixtures with difference in size, shape or density. In mixed traffic flow, slow vehicle and fast vehicle, as two types of particles, have different desired speed. We investigate the segregation along the road in mixed traffic flow by using a symmetric two-lane cellular automata model. A parameter D, which quantifies the degree of segregation, is defined. We study the density dependency of the parameter at different randomization probability. Simulation results show that segregation is more obviously in free flow region. We argue that the overtaking maneuvers have similar effect as percolation in granular flow.

PACS. 05.50.+q Lattice theory and statistics – 02.50.Ey Stochastic processes – 64.75.+g Solubility, segregation, and mixing phase separation – 89.90+n Other topics in areas of applied and interdisciplinary physics

1 Introduction

Nowadays cellular automata (CA) has become an excellent tool for simulating real traffic flow, because its efficient and fast performance when used in computer simulations [1,2]. In CA models, each vehicle is deemed as a particle, and the nature of the interactions among these vehicles is determined by the updating rule. In 1992, Nagel and Schreckenberg proposed the well-known Nagel-Schareckenberg (NaSch) model [3] for single-lane traffic flow. Although it is very simple, the NaSch model can reproduce some real traffic phenomena, such as the occurrence of phantom traffic jams and the realistic flow-density relation (fundamental diagram). The NaSch model is a minimal model in the sense that any further simplification of the model leads to unrealistic behaviors. But in single-lane CA models only one type of vehicle is considered, so it can not well describe the real traffic which is always mixed. Later, Chowdhury et al. [4] first investigated the mixed traffic system in two-lane model by introducing lane changing rules. Then overtaking maneuvers which performed by fast vehicle when hindered by slow vehicle were brought into CA models. In recent years, much attention have been payed to study the complex dynamics in mixed traffic by using two-lane CA models [5–12].

Granular mixtures exhibit a variety of intriguing behaviors [13]. Poured grains can sediment into regular striations [14]; shaken grains can spontaneously assemble into intricate and localized patterns [15,16]; and tumbled grains can generate radial, axial, and dynamical segregation patterns [17–19]. Small differences in size, shape, or density lead to flow-induced segregation, a phenomenon without parallel in fluids. Segregation of granular materials is a complex and imperfectly understood phenomenon.

In mixed traffic flow, fast vehicles and slow vehicles are two types of particles with different maximum speed. Nagatani first investigated the segregation between lanes by introducing asymmetric lane-changing rules [20]. In this paper, we investigate the segregation along the road in mixed traffic flow by using a symmetric two-lane CA model, and the configurations of the road and time evolution images are plotted to observe such effect. We also define parameters to quantify the degree of segregation. Simulation results show that segregation effects in free flow region are more obvious than that in congested flow region. Mechanisms of segregation effect are also studied. We proposed that the overtaking maneuvers change the relative positions of fast vehicle and slow vehicle, and this has similar effect as percolation in granular flow. So segregation in mixed traffic flow occurs.

Mixed traffic system has also been studied in the model of one-dimensional asymmetric simple exclusion process (ASEP) [21–23], in which each particle is assigned with a random hopping probability. The particles with lower hopping probability are deemed as slow particles, and the particles with higher hopping probability correspond to fast particles. If the ratio of slow particles is small in the

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system, phase separation will happen, which may be viewed as segregation phenomenon. The fast particles will form platoons behind the slow particle, and large gaps appear in front of the slow particle. In CA models for single lane traffic, such behavior will also occur if only a few slow vehicles exists in the system. In this paper, the ratio of slow vehicles is quite large in the system we studied (larger than 0.1), so phase separation does not happen in single lane model, segregation occurs only when lane changing and overtaking are possible.

This paper is organized as follows: in Section 2, we describe the model for the two-lane traffic. The method to transform the configurations of the road into time evolution image is proposed in Section 3. The parameter d quantifying the degree of segregation is defined in Section 4. Next, the simulation results are reported in Section 5. Finally, the conclusion is given.

2 Two-lane traffic model

In this work, we use the well-known basic cellular automata NaSch model for modelling the forward movements of the vehicles. Next, we briefly recall the definition of the NaSch model. The NaSch model is a discrete model for traffic flow. The road is divided into L cells, which can be either empty or occupied by a vehicle with a velocity $v = 0, 1, ..., v_{max}$. The vehicles move from the left to the right on a lane with periodic boundary condition. At each discrete time step $t \rightarrow t+1$, the system update is performed in parallel according to the following four sub-rules: (1) acceleration: $v_n \rightarrow \min(v_n + 1, v_{max})$; (2) deceleration: $v_n \rightarrow \min(v_n, d_n)$; (3) randomization: $v_n \rightarrow \max(v_n - 1, 0)$ with probability p; (4) position update: $x_n \rightarrow x_n + v_n$. Here v_n and x_n denote the velocity and position of the vehicle n respectively; v_{max} is the maximum velocity and $d_n = x_{n+1} - x_n - 1$ denotes the number of empty cells in front of the vehicle n ; p is the randomization probability.

This set of rules control the forward motion of vehicles. In order to extend the model to multi-lane traffic, one has to introduce lane-changing rules, which control the parallel motion of vehicles. So in multi-lane models the update step is usually divided into two sub-steps: in the first substep, vehicles may change lanes in parallel according to lane changing rules and in the second sub-step the lanes are considered as independent single-lane NaSch models.

In this work, we adopt the symmetric lane changing rules, in which both lanes are treated equally. Many lane changing rules can be found in the literature where a variety of rules are displayed [5–12]. However, all lane changing rules consist of two different parts: the security criterion (is it safe to change lane?) and the incentive criterion (is there a good reason to change lanes?). Here we investigate the symmetric model proposed by Chowdury et al. The rules can be described as follows:

$$d_n < \min(v_n+1, v_{max}) \ , \ d_{n,other} > d_n \text{ and } d_{n,back} > v_{max}. \quad (1)$$

Here $d_{n,other}, d_{n,back}$ denote the number of free cells between the nth vehicle and its two neighbor vehicles on the other lane at time t. $d_{n,back} > v_{max}$ is the security criterion and $d_n < \min(v_n + 1, v_{max})$, $d_{n,other} > d_n$ is the incentive criterion.

The segregation effects are studied in mixed traffic, so two kinds of vehicles are considered in the simulation, the fast vehicle with maximum speed $v^f_{max} = 5$ and the slow vehicle with maximum speed $v^s_{max} = 3$. We denote the ratio of slow vehicles to all vehicles as R.

3 Image acquisition

The digital image obtained in the experiment is usually analyzed to study the segregation effect of granular mixtures. Here the configurations of the two-lane road are transformed into image by using the method below: the two-lane road is divided into m parts, and every part contains $2l$ ($l = \max(2 \times 1.0/\rho, l_0)$) cells, so $m = L/l$. At small densities, we choose larger l so that there are always vehicles in every part of the road. At larger densities, small l is selected but not less than l_0. In each part, assume that there are N_s slow vehicles and N_f fast vehicles, then the composition of the part is represented by a real number $c = (N_f - N_s)/(N_f + N_s)$. So $c = 1.0$ denotes that all the vehicles in the part are fast vehicles; reversely $c = -1.0$ means that all the vehicles in the part are slow vehicles. If there are no vehicles in the part, c is set to zero. We can obtain a one dimensional image with m pixels length and the deepness of the color is determined by the value $(c+1)/2$, which is between 0 and 1. During the evolution of traffic flow, we can obtain a time evolution image of the configurations of the road.

4 The parameters quantifying the degree of segregation

In two-lane CA model, the vehicles drive in discrete cellular space, so the positions of each vehicle can be measured at each time step. For each vehicle, there is a neighbor region on the two-lane road. This region also has a length of l which is dependent on the density and the same as that value in Section 3. The vehicle is at the center of the region. We define the percent similar as the proportion of the same type of vehicles in the neighbor region to each vehicle. The average percent similar of all the slow (fast) vehicles is defined as the degree of segregation of slow (fast) vehicles represented by D_s (D_f). And the degree of segregation of the system is D, which is the average percent similar of all the vehicles. We argue that the larger the value of D is, the more obvious the segregation effect is.

5 Simulation results

We carry out the simulations of the two-lane traffic model mentioned in Section 2 on a lattice of $2L$ sites with periodic boundary condition. In the initial condition, slow

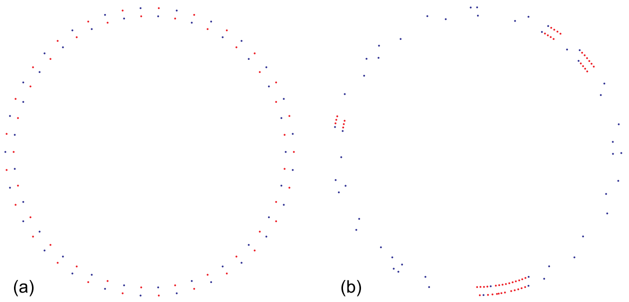

Fig. 1. At small density $\rho = 0.05$, the configurations of the two-lane road (a) in the initial state; (b) at 2000 time step. The parameters are $R = 0.5$ and $p = 0.05$. The segregation effect appears quickly and after 2000 time step the configuration of the road changes little.

vehicles and fast vehicles are well mixed on the two-lane (Figs. 1a and 3a). Since periodic boundary condition is considered, the two lanes are taken as two circles. As we can see in Figures 1 and 3, the inner circle represents the left lane while the outer circle corresponds to the right lane. The blue points correspond to the slow vehicles and the red points to the fast ones. They drive anticlockwise.

We first study the case $R = 0.5$. Both the configurations of the road and the time evolution images are shown to see the segregation effect. The parameter $l_0 = 10$ is used. The road length $L = 1000$ is selected when drawing the configurations of the road while $L = 10\,000$ is used in making the time evolution images. The segregation effect can be seen more clearly by doing those measures. Simulation results indicate that L does not influence the evolution process of traffic flow.

Figure 1 shows the results at density $\rho = 0.05$. When fast vehicles are hindered by slow ones, they will try to overtake slow ones. But slow vehicles may form "plug" from time to time, then fast vehicles have to drive behind the "plug" until it dissolves. Such process can be clearly seen in Figure 2. In the beginning, the color is gray, which means the slow vehicles and the fast vehicles are well mixed. After about 200 time steps, some "plugs" are formed on the road. Behind the "plugs", fast vehicles assemble and a narrow white area appears. In front of the "plugs", fast vehicles continue overtaking slow vehicles and leave them behind, so black area expands. As time proceeding, the segregation effect becomes more obvious and the gray region gradually disappears.

At $\rho = 0.2$ both fast vehicles and slow vehicles are segregated into bands along the road in the stationary state, but to reach the state takes a much longer evolution time. The configurations of the road at different time step ($t_0 = 0, t_1 = 10\,000, t_2 = 100\,000$ and $t_3 = 200\,000$) are shown in Figure 3. At time step t_1, small bands are formed along the road; at time step t_2, small bands merge into large bands; and there is only a little difference between the configuration of the road at time step t_2 and that at time step t_3. This indicates that the coarsening behavior, which is commonly exist in disordered exclusion models [23], occurs. The time dependence of the degree

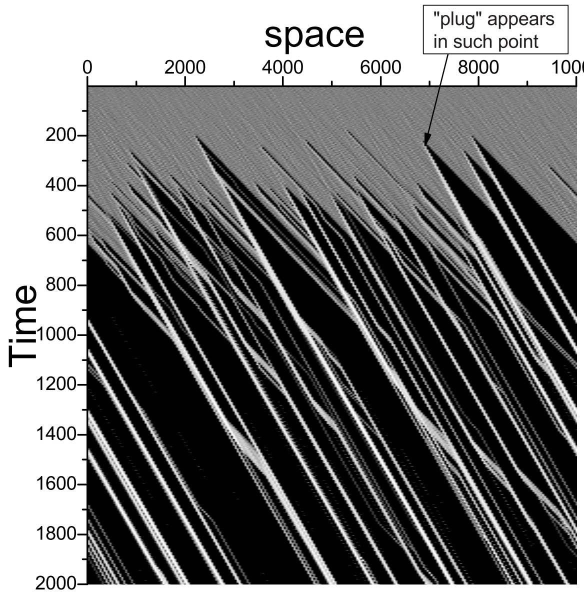

Fig. 2. The time evolution image of the configuration of the road at $\rho = 0.05$ (thus $l = 40$) during the time step from 0 to 2000. The parameters are $R = 0.5$ and $p = 0.05$. Black corresponds to $c = -1$, and white represents $c = 1$. Each black area corresponds to a "plug", so the number of "plugs" is small at small density.

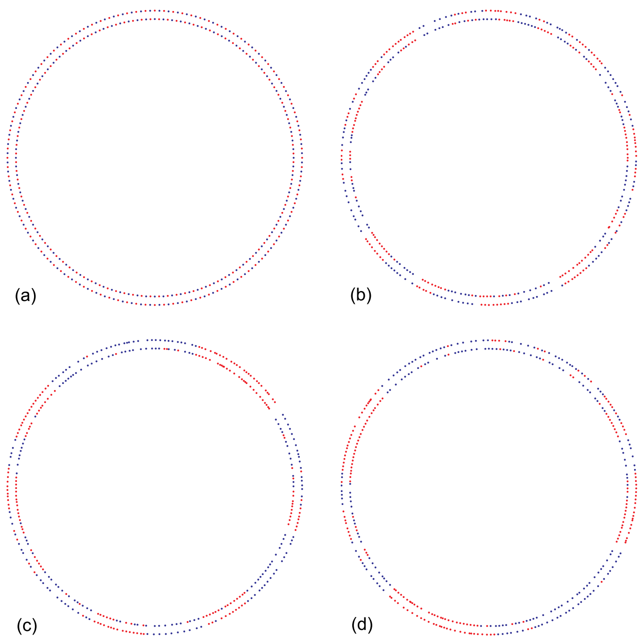

Fig. 3. At $\rho = 0.2$, the configurations of the two-lane road. (a) At $t_0 = 0$ time step (the initial state); (b) At $t_1 = 10\,000$ time step; (c) at $t_2 = 100\,000$ time step; (d) At $t_3 = 200\,000$ time step (the stationary state). The parameters are $R = 0.5$ and $p = 0.05$. As time increasing, segregation effect becomes more obvious, and the evolution time to reach a stationary state is much longer than that at $\rho = 0.05$.

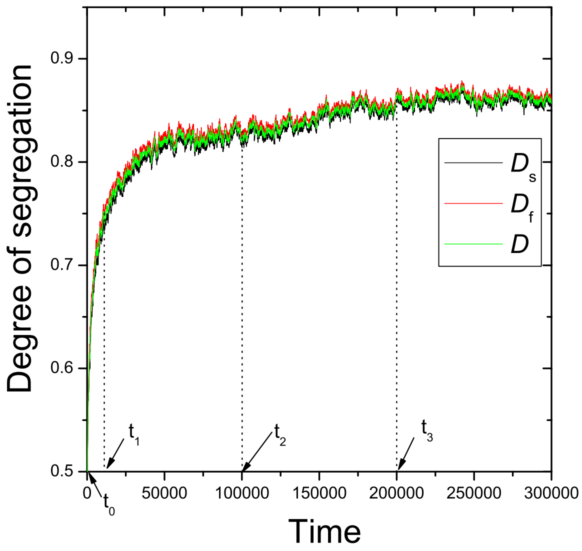

Fig. 4. The plot of degree of segregation against time steps. The parameters are $\rho = 0.2$, $R = 0.5$ and $p = 0.05$.

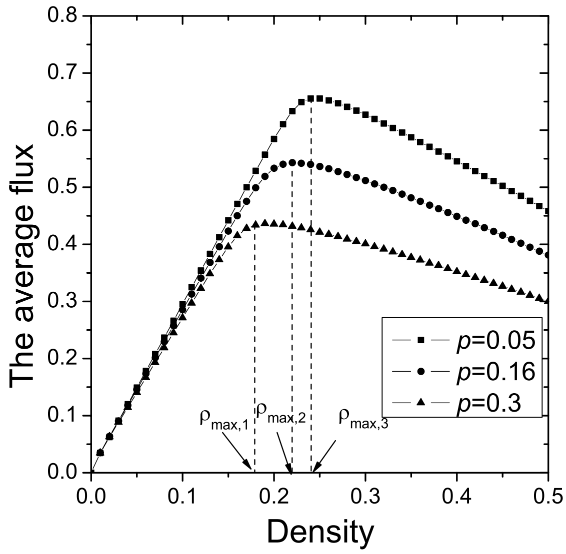

Fig. 6. The fundamental diagrams of the two-lane CA model in the case of $R = 0.5$. At the density range $\rho < \rho_{max}$ the traffic state is free flow ; beyond ρ_{max} traffic jam exists. Here $\rho_{max} = \rho_{max,1}$, $\rho_{max,2}$ and $\rho_{max,3}$ when $p = 0.05$, 0.16 and 0.3, respectively.

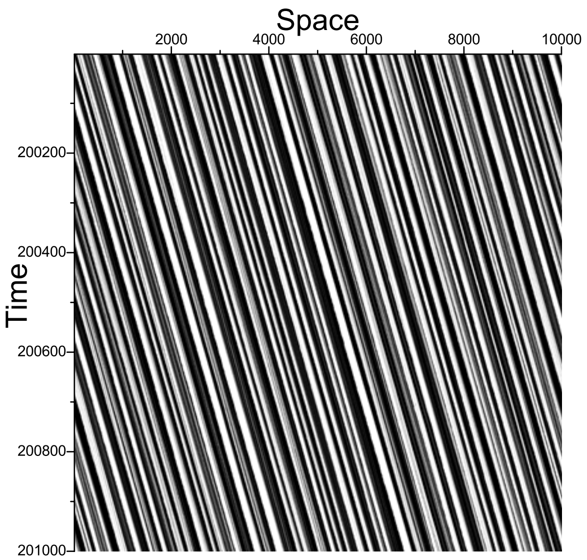

Fig. 5. The time evolution image of the configurations of the road at $\rho = 0.2$ ($l = 10$) during the period from 200 000 to 201 000 time step. The parameters are $R = 0.5$ and $p = 0.05$. The segregated black and white bands indicate the segregation of mixed traffic flow.

of segregation is shown in Figure 4. The values of D_s, D_f and D are only slightly different, and they are 0.5 in the initial state but reach a value of 0.87 in the stationary state. They first increase quickly, then increase slightly, and at last approximately maintain at a constant after about 200 000 time step. Compared to Figure 3, it can be seen that the parameters D_s, D_f and D can really reflect the degree of segregation. In addition, the time evolution image of the configurations of the two-lane road during a period of 1000 time steps in stationary state is shown in Figure 5. The white and black bands indicate a well segregation of fast and slow vehicles.

Those results indicate that the segregation effect also exists in mixed traffic flow because fast vehicle and slow vehicle can be deemed as two kinds of particles with different maximum speed.

In CA models for traffic flow, density ρ and randomization probability p are two important parameters (Fig. 6). As the density increasing, traffic flow first performs free flow (i.e., traffic in free flow region). When the critical density is exceeded, traffic is in congested flow region and jams can occur spontaneously out of homogeneous traffic. The randomization p has great influence on the lane-changing frequency in two-lane CA model. So we next investigate the influence of the parameters ρ and p on the degree of segregation in the case of $R = 0.5$. The first 200 000 time steps are discarded, the value of D is obtained by averaging the last 100 000 time steps.

First, we study the difference between D_s and D_f at different densities when $p = 0.05$ (inset figure in Fig. 7). D_f is a little larger than D_s when $\rho < 0.2$. This is because at small densities, fast vehicles are much more condensed than slow vehicles (Fig. 1). They almost equal to each other when $\rho \geq 0.2$. Since the difference is small, next we mainly focus on analyzing the value of D. The density dependence of D with different randomization probability p is drawn in Figure 7. It shows that D first decreases to a local minimum value then grows to a local maximum

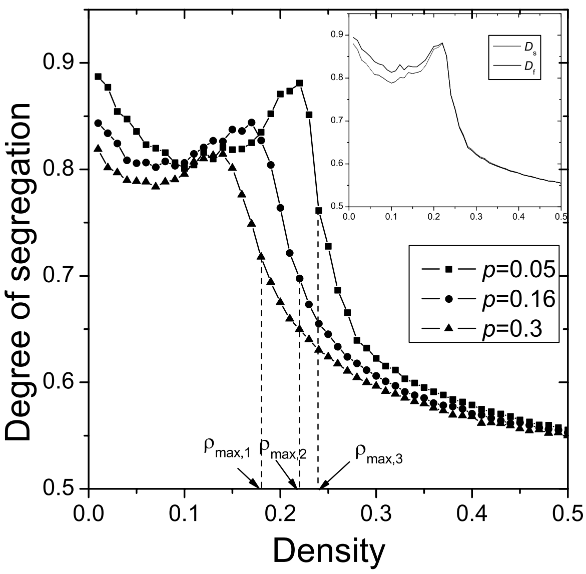

Fig. 7. The value of D as a function of density ρ in the case of $R = 0.5$. The inset figure shows the difference between D_s and D_f when $p = 0.05$.

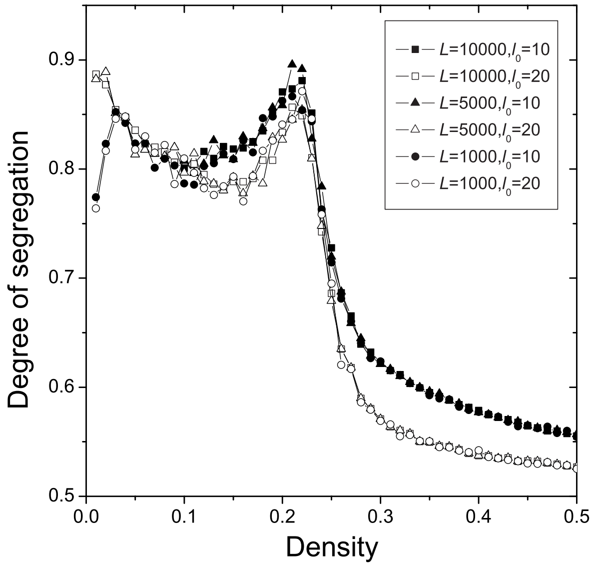

Fig. 8. The value of D as a function of density with different l_0 and road length L. The parameters are $R = 0.5$ and $p = 0.05$.

value and as the density increasing to congested flow region, D falls to small values which is still slightly larger than the initial value of 0.5. Figure 7 also shows that the values of D perform the same trend at different p. We can conclude that the segregation occurs in free flow region, but it is greatly suppressed in congested flow region. As p decreasing, the transition from free flow to congested flow occurs at a larger density, accordingly the critical density corresponding to the local maximum value of D increases.

We know that granular segregation can be driven by percolation, where smaller particles pass through the holes created by the larger particles [16]. In two-lane CA models for mixed traffic, fast vehicles will try to overtake slow ones when they are hindered. Overtaking maneuvers can change the relative positions of fast vehicles and slow vehicles. We argue that overtaking maneuvers have similar effect as percolation in granular flow.

We also investigate the evolution of degree of segregation in single lane case. It is found D, D_s, and D_f remain unchanged with time. This confirms that the segregation is caused (or at least can be enhanced) by overtaking.

Next the influence of randomization p on the degree of segregation is discussed.

- At small densities ($\rho < 0.1$), the average life time of "plugs" becomes longer with the decrease of p. Therefore, the degree of segregation increases with the decrease of p.
- With the increase of p, the transition from free flow to congestion flow occurs at a smaller density. As shown earlier, when congestion occurs, the segregation effect is greatly suppressed. Therefore, the degree of segregation decreases with the increase of p at large densities.
- In the intermediate density range $0.1 < \rho < 0.18$, the variation of the degree of segregation D with p is quite complex. This is a combination effect due to, on the one hand, life time of a small "plug" decreases with the increase of p, and on the other hand, two small plugs may have more chances to merge into one large plug and the life time of a large plug is much larger than a small one.
- In the special case of $p = 0$, segregation does not happen if initially the fast vehicles and slow vehicles are well mixed. This is because (i) if $\rho > 0.11$, then any successive two slow vehicles form a plug and the plug will not dissolve [10]. Therefore, the fast vehicles have no chance to overtake. Consequently, no segregation happens; (ii) if $\rho < 0.11$, then no plug will form. As a result, the segregation does not happen, either.

Now, we investigate the influences of the parameter l_0 and the road length L on the segregation effect. The results are shown in Figure 8. We can see that the value of D in case of $l_0 = 20$ is smaller than that in case of $l_0 = 10$ when $\rho > 0.1$. This is explained as follows. Suppose that $l = 1$ is selected, the neighbor region of each vehicle usually contains only itself, so D keeps a large value in all cases. D is constant with the value of 0.5 if $l = L$. This means larger value of l reduces the value of D. In order to quantify the degree of segregation, an intermediate value of l is selected. The road length L does not effect the segregation effect, except that D decreases at very small densities when $L = 1000$ because there are so few slow vehicles in the system and the "plug" rarely forms.

In real traffic system, the ratio of slow vehicles is usually small. Now we investigate the case of $R = 0.1$. The configurations of the road and the time evolution image

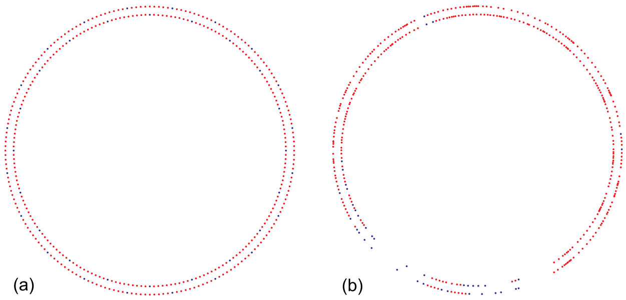

Fig. 9. The configurations of the two-lane road in the case of $R = 0.1$, $\rho = 0.2$ and $p = 0.05$. (a) The initial state; (b) the stationary state (after 200 000 time steps).

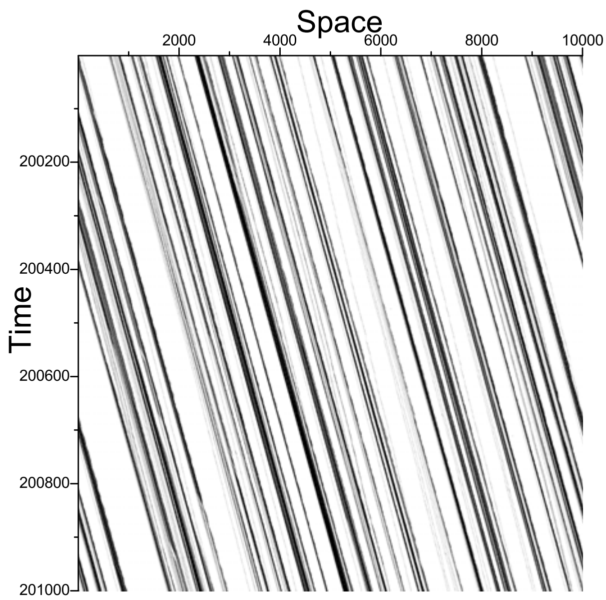

Fig. 10. The time evolution image of the configurations of the road in a time period of 1000 during stationary state. The parameters are $R = 0.1$, $\rho = 0.2$ ($l = 10$) and $p = 0.05$. Narrow black bands indicate that slow vehicles drive together in stationary state.

are drawn in Figures 9 and 10 respectively. Figure 10 shows that slow vehicles form narrow bands along the road. In the initial state, slow vehicles are encapsulated by fast vehicles, and $D_s = 0.1$, $D_f = 0.9$. But in the final state, D_s can reach high values and D_f is larger than 0.9 in free flow region (Fig. 11). Although the ratio of slow vehicles is small and the slow vehicles are well separated, they can drive together and form "plugs" in the final state. Then the fast vehicles will be hindered by those "plugs" and the average flux in the case of $R = 0.1$ is almost the same as that in the case of $R = 1.0$ [Fig. 12]. Furthermore, smaller value of p can enhance the degree of segregation in the whole density range, because the number of slow vehicles is small and the small plugs are far from each other and has no chance to merge.

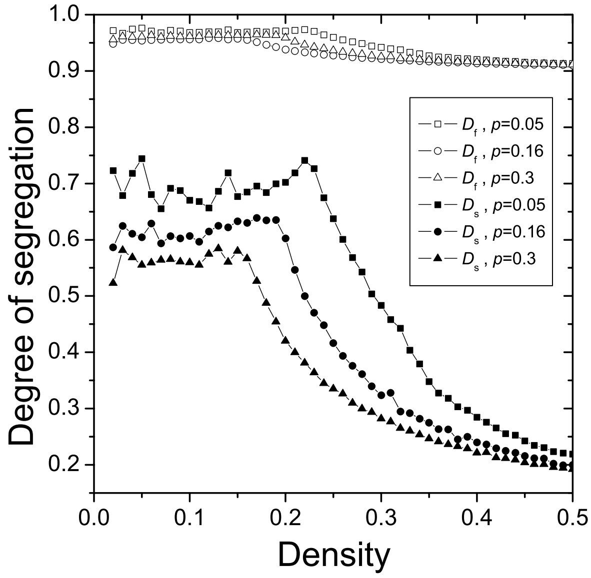

Fig. 11. Comparison of the values of D_s and D_f as a function of density ρ in the case of $R = 0.1$ at different p.

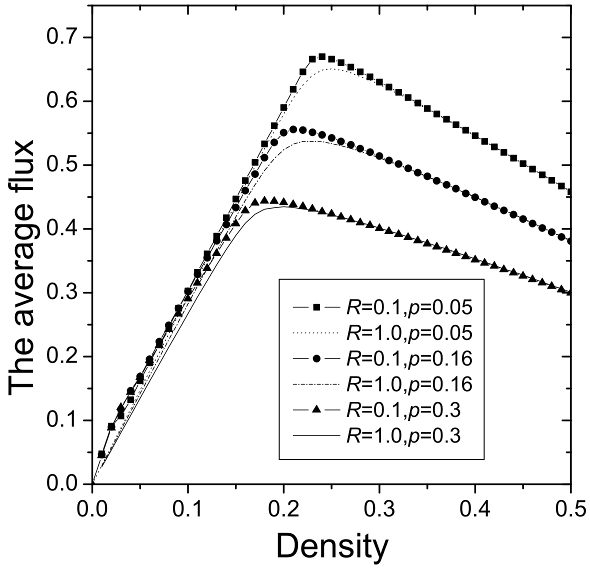

Fig. 12. Comparison of fundamental diagrams with different R and p.

6 Conclusion

In mixed traffic flow, the segregation effect also exists. As two kinds of particles, slow vehicles and fast vehicles segregate into bands along the road. We investigate such effect in the symmetric two-lane CA model for mixed traffic flow. In order to quantify the degree of segregation effect, we define the parameters D_s, D_f and D. Simulation results indicate that the values of those parameters can really reflect the degree of segregation effect.

In case of $R = 0.5$, D_f is a little higher than D_s in the density region $\rho < 0.2$, but there is almost no difference when $\rho \geq 0.2$. As the density increases, the value of D first decreases then increases; after a local maximum value is reached it quickly reduces to low values.

In case of $R = 0.1$, although slow vehicles are well separated in the initial condition, they still can drive together and form "plug" in the final state. This is the reason why the average flux is almost the same as that in case of $R = 1.0$.

We argue that the mechanism of segregation in mixed traffic flow is overtaking maneuver and it has the similar effect as percolation in granular flow.

This work is partially supported by National Basic Research Program of China (2006CB705500), the National Natural Science Foundation of China (NNSFC) with Grant Nos. 70471088, 70501004, 10532060, 10404025, the National Outstanding Young Investigator of NNSFC through Grant No. 70225005, the Special Program for the Preliminary Research of Momentous Fundamental Research of Ministry of Science and Technology of China through Grant No. 2005CCA03900, the School Foundation of Beijing Jiaotong University through Grant No. 2004SM018 and the 211 Project of Discipline Construction of Beijing Jiaotong University.

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